

## **DIAGNOSING THE JUNIOR HIGH SCHOOL STUDENTS' DIFFICULTIES IN LEARNING MATHEMATICS**

Dr. Heri RETNAWATI  
Yogyakarta State University  
Mathematics and Science Faculty  
Yogyakarta- INDONESIA

### **Abstract**

The improvement of learning quality might be pursued by improving the learning process based on the results of diagnosing the students' learning difficulties. The study then was to diagnose the junior high school students' learning difficulties in completing the mathematics test items of the national examination. The study is a quantitative descriptive study. The diagnosis of learning difficulties was conducted by performing the cognitive diagnostic assessment through using DINA model. The data that had been used in the study were students' responses of math test of national examination of junior high schools located in the Province of Yogyakarta Special Region, Indonesia. The students' difficulties were detected by implementing the CDM package provided by the R program and using the output of the latent classes were identified. The results of the study showed that the students experienced some difficulties in completing the national examination test in many items on number, geometry and statistics, and understanding the narrative text items.

**Keywords:** Diagnostic, Learning Difficulty, Attribute, DINA.

### **INTRODUCTION**

In an educational system, assessment has been an urgent activity that should be performed. Educational assessment has multiple advantages and one of these advantages namely student evaluations (summative and formative), instructional decisions, selection, placement and classification decisions, policy decisions and counselling and guidance decisions (Reynolds, Livingstone, Wilson, 2010). The specific advantage in relation to understanding the students' learning difficulties, with regards to these advantages, is known as diagnostic. Nitko & Brookhart (2007) state that there are six approaches of diagnostic assessment in relation to the learning problems namely: (a) the approach of strengths and weaknesses on the capacity of a subject; (b) the approach of identifying the prerequisite knowledge weaknesses; (c) the approach of identifying the learning objectives that have not been mastered; (d) the approach of identifying the students' mistakes; (e) the approach of identifying the students' knowledge structure; and (f) the approach of identifying the competencies for completing the narrative test items.

In order to perform such diagnostics, a teacher might perform a test. By performing a test, a teacher might gather accurate regarding information about the concept errors that the students have based on those information (Mehrens & Lehmann, 1984) and might identify multiple difficulties that occur and develop in the past or the present learning process (Thorndike & Hagen, 2005). The test should be specially designed by paying attention to the competencies of the materials that the students consider being difficult (Grondlund, 2011).

In order to identify the students' difficulties, there are several models that a teacher might implement. In relation to these models, the Cognitive Diagnostic Models (CDMs) have been a psychometric model that measure the students' mastery in the cognitive dimension and the mastery or the capacity that will be assessed is the incapability-weakness or the capability-strength of the students toward certain competencies (Chen, et al., 2015). Then, in order to understand the students' difficulties, a teacher should define first the attributes that will be measured on the test that will be

developed or be administered. These attributes have been the knowledge description in completing the tasks for certain domains (Gierl, 2011) and the fundamental cognitive process that will be necessary for completing the test (Zheng et al., 2008; Zhou et al., 2009). In mathematics, the attributes consist of 3 categories namely the content attribute (content-related knowledge and skills), the process attribute (the mathematical thinking skills) and the skill attribute (the special skills unique to item types) (Tatsuoka, 2009).

One of the cognitive models that belong to the form of Cognitive Diagnostic Assessment (CDA) is the Attribute Hierarchy Method (AHM). AHM has been a psychometric method that classifies the item responses into a set of structured attribute patterns in relation to the competencies that are different from the other cognitive model through a task that has been given to the students (Leighton et al., 2007). The cognitive model contains the attributes that have been set as a description of procedural knowledge or the statements that the test participants will need in answering the test items correctly (Leighton & Gierl, 2007; Robert & Gierl, 2010).

The attributes might be defined for each response pattern under observation so that the attributes will provide detail information regarding their performance toward the learning participants (Wang, 2005). There are four forms of fundamental structure that compose a stratified network of the attribute, namely the linear form, the convergent form, the divergent form and the unstructured form (Gierl, Leighton & Hunka, 2007). Next, by viewing the inter-attribute relationship, the next step will be grouping the materials based on the competencies which attributes will be defined, by defining the attributes that will be measured in each item and by designing the attribute hierarchy into a group of items. The attribute hierarchy then will be used for designing the Q-Matrix that has been developed by Tatsuoka (2009). Q-Matrix is a matrix with  $m$  rows and  $n$  columns and the elements of the matrix consist of 0 and 1. The cognitive capacity and the other necessary capacities in answering the test items by means of the column will be represented by these attributes while the line represents the items. The attributes in an item is the prerequisite materials that the students should master in completing each test item.

There are several that might be selected and be implemented into the AHM. These models are, namely, the compensatory model and the non-compensatory model. In the compensatory model, the students' incapability to complete one attribute might be compensated by the other attributes; meanwhile, in the non-compensatory model all of the attributes should be mastered by the students in order attain the correct answers. The non-compensatory model consists of DINA, NIDA and NC-RUM Model. On the other hand, the compensatory model consists of DINO, NIDO and C-RUM Model (Rupp, Templin & Henson, 2010). There are also some experts who have defined a fusion model (Aryadoust, 2011). In this, the researcher will only implement the DINA (Deterministic-Input, Noisy and Gate) Model that is appropriate to the test that will be analysed. In addition, the model has been selected because, based on the results of a study by Basokcu, Ogretmen & Kelecioğlu (2013), the research found "DINA model could give rather better results to estimate student profile in tests where higher level and progressive behaviours are used together."

In the DINA Model, the deterministic input is the first element. The deterministic input takes the form of latent variables in the items and the respondents. The next element is the and-gate, which shows that the students might complete all test items appropriately if they master all of the attributes in those items. Then, the third element is the noise, which takes the form of slip and guessing parameter. A slip occurs if the students master the overall capacity to complete each test item politely but they slip and complete the test items inappropriately. On the other hand, a guessing occurs if the students at least master one of the capacities for completing the test items but they might guess or answer the test items appropriately (de la Torre, 2009). Within a good model, the good slip and guessing parameter should be small. Such criterion is appropriate with the opinion of Rupp, Templin & Henson (2010) that "... the postulated DINA Model is attained when the estimated of the slipping and guessing parameters are small". By using the slip and guessing parameter a teacher might estimate the item differential capacity. Furthermore, by using the estimation a teacher might

generate the percentage of latent class that will be implemented for designing the interpretations and the attributes that the students have not mastered.

Several studies have been conducted in relation to the diagnostic assessment. Sun & Suzuki (2013) implemented a DINA-type CDA for diagnosing the students' difficulties in learning the fraction. The profile of each student was described by using the web diagram. Huebner (2010) tried to develop the CAT cognitive diagnostic. Romero, Ordonez & Ponsoda (2014) detected a misspecification of Q-Matrix. Kusaeri (2014) developed a diagnostic test for the 8<sup>th</sup> grade Mathematics learning materials and he had been able to identify the students' mistakes in relation to the arithmetic, the verbal-type test items and the basic concept of algebra.

In Indonesia, the students in the final stage of each educational degree should participate in the national examination. The national examination is administered in order to assess the achievement of graduates' competencies that will be used for mapping the program and/or the educational unit quality; for serving as the basis of selection in the higher educational degree; for determining the graduation of learning participants from the program and/or the educational unit; and for developing and providing assistance toward the educational units within their efforts of improving the educational quality (Peraturan Pemerintah No. 19 Tahun 2005). In the national examination, the students' competencies are measured based on certain indicators. One of the advantages that might be gained from administering the national examination is that the results of national examination might be used for mapping the learning capacities and difficulties. By understanding the mapping of learning capacities and difficulties, the teacher might improve the learning process in the upcoming years. With regards to this aspect, the identification of learning difficulties based on the national examination results should be conducted in order to improve the learning process toward the aspects of teaching method, assessment manner, learning implementation decision and learning implementation assessment in the school. The study diagnoses the junior high school students' difficulties in learning Mathematics by using the national examination results. By identifying the students' difficulties in learning Mathematics, the researcher might improve the learning process both in the effort of improving the students' capacity individually and in the effort of improving the class-level or the school-level competency.

## METHODS

The study was a descriptive explorative research with a quantitative approach. According to the objective of the study, the data that would be implemented in the study were all responses provided by the Mathematics National Examination participants who completed the Package 1 Test Items from the junior high school degree in the Province of Yogyakarta Special Region in Indonesia. The number of test participants was 1,445 people and the number included all of the test participants from both the state and the private junior high schools. The data were gathered by means of documentation from the national examination administration that had been attained from the Ministry of Education and Culture, the Republic of Indonesia. The test items consisted of 40 multiple choices with dichotomous scoring process (if the test item was answered correctly then the student would gain 1 as the score, but if the test was answered incorrectly then the student would gain 0 as the score).

The steps of diagnosing the students' difficulties in learning the national examination test items referred to the steps of analysis through the CDM implementation that had been suggested by Ravand & Robitzch (2015) namely: designing the attribute specifications, analysing the test items and viewing their relationship to the Q-Matrix, establishing the inter-sub-skills relationship and mastering the test participants' mastery capacity by means of DM. According to these steps, the 40 national examination test items would be classified based on the materials. The materials included the rational numbers and the integers, the exponential number, the rows and lines, the linear equation system, the sets, the function and straight line, the planes, the triangle characteristics, the circular tangent, the solids and the statistics and probability. Based on these materials, the researcher designed 11 Q-Matrixes based on the inter-sub-skills relationship that the students demanded to complete the test

items appropriately. Then, the researcher also grouped the students' responses toward the items that had been in accordance with the Q-Matrix. The further analysis was conducted by implementing the CDM package from the R program that generated the slip and guessing parameter and the latent attributes. Next, these attributes would be interpreted and the interpretation included understanding the sub-skills that had been relatively difficult in comparison to the other sub-skills within a material group based on the attributes within the Q-Matrix.

## FINDINGS

After the researcher performed an analysis by means of CDM package from the R program toward each Q-Matrix attained from the Mathematics National Examination Test Items, the researcher found the noise parameter of the DINA Model. These parameters were presented in Table 1.

Table 1: The Parameter Values of DINA Model

Item	Guess	Slip	d	Item	Guess	Slip	d
1	0.267	0.030	0.703	21	0.088	0.180	0.732
2	0.296	0.064	0.640	22	1.000	0.000	0.000
3	0.145	0.104	0.751	23	0.205	0.195	0.600
4	0.036	0.563	0.401	24	0.182	0.140	0.678
5	0.000	0.740	0.260	25	0.067	0.171	0.762
6	0.200	0.036	0.764	26	0.095	0.179	0.726
7	0.148	0.057	0.795	27	0.155	0.042	0.803
8	0.238	0.049	0.713	28	0.315	0.020	0.665
9	0.154	0.073	0.773	29	0.132	0.131	0.737
10	0.227	0.041	0.732	30	0.195	0.084	0.721
11	0.136	0.041	0.823	31	0.110	0.080	0.810
12	0.125	0.042	0.833	32	0.697	0.002	0.301
13	0.089	0.061	0.850	33	0.280	0.014	0.706
14	0.082	0.151	0.767	34	0.102	0.076	0.822
15	0.240	0.022	0.738	35	0.077	0.113	0.810
16	0.047	0.246	0.707	36	0.430	0.018	0.552
17	0.138	0.106	0.756	37	0.115	0.213	0.672
18	0.191	0.042	0.767	38	0.212	0.092	0.696
19	0.112	0.209	0.679	39	0.182	0.030	0.788
20	0.117	0.133	0.750	40	0.325	0.041	0.634

Based on the table, the test items that had the lowest discriminative index ( $d$ ), according to Table A.1, were the item number 22, number 5 and number 32. From the three items, the one that had the lowest discriminative index was item number 22 namely 0.000. Since the discriminative index of number 22 had been equal to 0.000, the researcher might imply that the item number 22 might not discriminate well the capable and the incapable students. In relation to both parameters of DINA Model, the cause of the low discriminative index in the item number 22 was the guessing parameter of item number 22 had the highest score namely 1.000. Since the guessing score had been equal to 1.00, the researcher might imply that all students had completed the test item number 22 not because of the fact that they had mastered all of the necessary attributes for completing the test item but because of the fact that they had guessed the answer correctly.

The item discriminative index of the item number 5 was equal to 0.260 and had been the second lowest capacity. The reason was that the slip value in the test item number 5 had been the highest one namely 0.740. Therefore, the researcher might imply that around 74.00% of the students who provided the wrong answer had been the students who had all of the necessary attributes for completing the test item correctly but they slipped themselves in the completion process. Then, the third item that had the lowest item discriminative index was the test item number 32 namely 0.301.

In the test item number 32, the guessing parameter was 0.697 and had been the highest guessing parameter after the test item number 22. As a result, the researcher might imply that 69.70% of the students had completed the test item correctly because they guessed the answer.

The further analysis was estimating the percentage of latent capacity for each attribute. The analysis was conducted toward each Q-Matrix in the attribute group for the concordant materials. In accordance with the number of the matrix, the researcher performed the interpretation for 11 times.

**The Integers and Fractions (Analysis for the Q1-Matrix)**

The Q1-Matrix was established by four test items namely the test item number 1, number 2, number 6 and number 21. From the four items, there were four attributes that the researcher found in the Q1-Matrix. The attributes that established the Q1-Matrix were namely: (A1) fraction operation; (A2) comparison; (A6) social arithmetic; and (A21) comparison in the form of narrative test items. For a better description, the researcher would display the items and the attributes that established the Q1-Matrix in the Table 2.

Table 2: The Items and the Attributes that Established the Q1-Matrix

Item Number	Attribute			
	A1	A2	A6	A21
1	1	0	0	0
2	0	1	0	0
6	0	0	1	0
21	0	0	0	1

There were four attributes that established the Q1-Matrix. From the four attributes, the possible number of latent class would be 16 classes. According to the results of analysis by means of CDM packages provided by the R program, the researcher attained the percentage for each class. In order to identify the difficulties in learning mathematics based on the attribute mastery in the Q1-Matrix according to the latent class percentage, the researcher would like to display Figure 1.

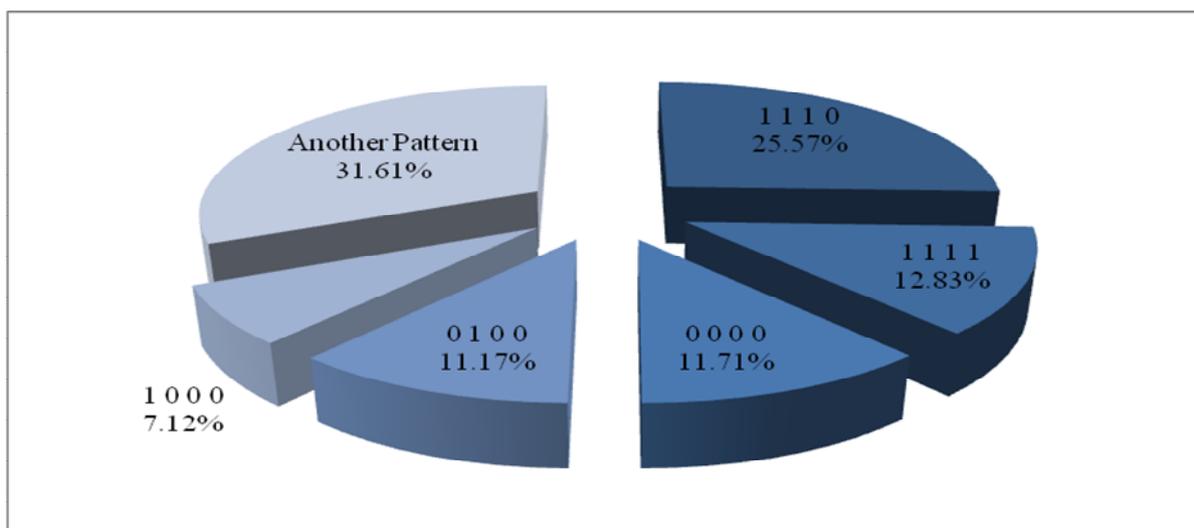


Figure 1: The Latent Class Percentage in Accordance with the Attributes Establishing the Q1-Matrix

Based on Figure 1, the researcher understood that the most dominant latent class percentage had been found in the latent class 1 1 1 0. The percentage of latent class 1 1 1 0 that had been equal to 25.27% showed that around 25.27% of the students had not mastered the attributes (A21) in responding to the comparison within the narrative test items. The A21 Attribute had been the necessary attribute for completing the test item number 21 correctly. In order to complete the test

item number 21 correctly, the students had to master the concept of comparison, to understand the narrative text items and to determine the results of fraction operation. Based on the most dominant percentage in the Figure 1, the researcher detected that most of the students had not mastered the capacity of understanding the narrative test items and determining the results of fraction operation because they had not mastered the concept of comparison.

### The Exponentials (Analysis for the Q2-Matrix)

The Q2-Matrix was established by three items namely the test item number 3, number 4 and number 5. From the three items, there were three attributes. The attributes that established the Q2-Matrix were (A3) exponentials operation, (A4) square root operation and (A5) square root operation in the form of rationalizing the denominator. The items and the attributes that established the Q2-Matrix might be viewed in the Table 3.

Table 3: The Items and the Attributes that Established the Q2-Matrix

Item Number	Attribute		
	A3	A4	A5
3	1	0	0
4	0	1	0
5	0	0	1

According to Table 3, there were three attributes that established the Q2-Matrix. The three attributes would generate 8 latent classes. With the assistance of CDM packages in the R program, the researcher might attain the percentage of attribute mastery for each latent class. In order to find the difficulties in learning Mathematics based on the attribute mastery within the Q2-Matrix and in accordance with the latent class percentage, the researcher would like to display the Figure 2.

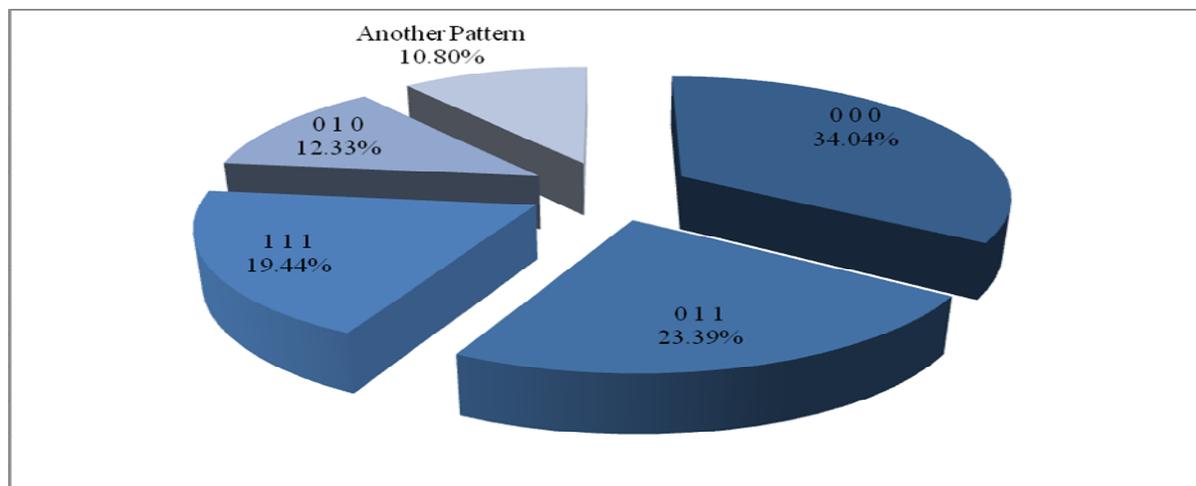


Figure 2: The Latent Class Percentage in Accordance with the Attributes Establishing the Q2-Matrix

Based on Figure 2, the researcher might understand that the most dominant latent class percentage had been found in the latent class 0 0 0. The percentage of latent class 0 0 0 that had been equal to 34.04% showed that around 34.04% of the students had not mastered the three attributes in the Q2-Matrix. The three attributes were found in each attribute from the test item number 3, number 4 and number 5. For the test item number 3, in order to provide the correct answer the students should have the capacity of determining the results of number exponents. Then, for the test item number 4 in order to provide the correct answer the students should have the capacity of determining the results of number exponents operation and of denominator rationalization. Last but not the least, for the test item number 5 in order to provide the correct answer the students should master the capacity of rationalizing the denominator that took the form of fraction. Therefore, the dominant

capacities that had not been mastered by the students were the capacity of determining the number exponent results, of determining the fraction operation and of rationalizing the square root-type denominator.

### The Number and Row (Analysis for the Q3-Matrix)

The Q3-Matrix was established by three items namely the test item number 7, number 8 and number 9. The three attributes that established the Q3-Matrix were (A7) arithmetic number, (A8) arithmetic row and (A9) arithmetic row by understanding the narrative text items. In order to understand the Q3-Matrix, the researcher would like to display the items and the attributes that established the Q3-Matrix in the Table 4.

Table 4. The Items and the Attributes that Established the Q3-Matrix

Item Number	Attribute		
	A7	A8	A9
7	1	0	0
8	0	1	0
9	0	0	1

The three attributes in the Table 4 generated 8 latent classes. The percentage of the 8 latent classes in the Q3-Matrix might be used for identifying which attribute that had or that had not been mastered by the students. Based on the results of analysis by means of CDM packages in the R program, the researcher attained the percentage of each latent class as having been displayed in the Figure 3 as follows.

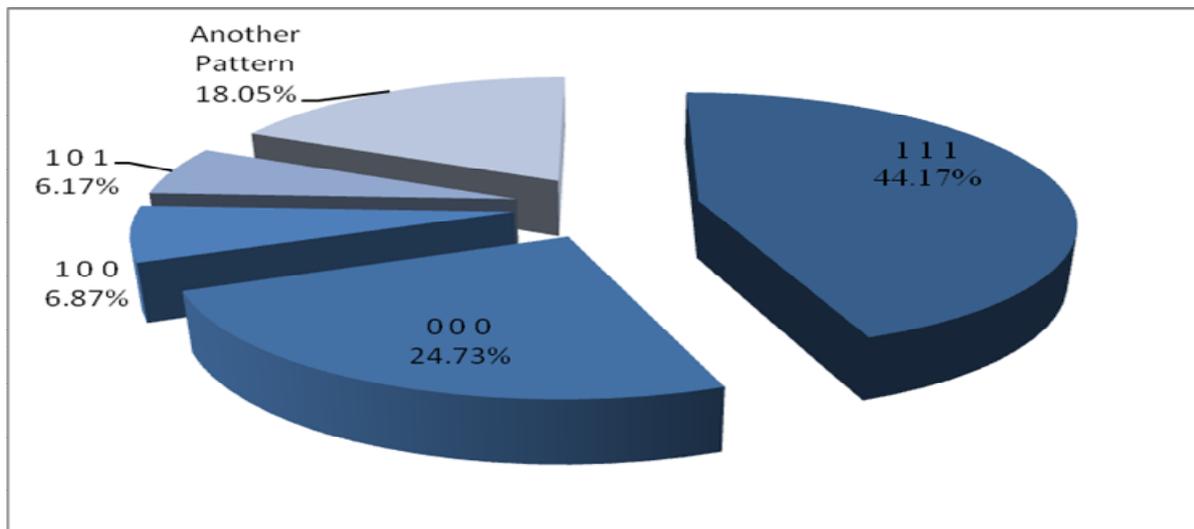


Figure 3: The Percentage of Latent Class in Accordance with the Attributes Establishing the Q3-Matrix

The most dominant latent class percentage shown in the Figure 3 was 44.17% and was found in the latent class 1 1 1. The dominant latent class percentage showed that 44.17% of the students had mastered all of the attributes that established the Q3-Matrix. Since 44.17% of the students had mastered the attributes, the remaining 55.83% students had not mastered at least one of the attributes that established the Q3-Matrix. From Figure 3, 24.73% of the students had not mastered all of the attributes in the Q3-Matrix. Therefore, most of the students had mastered the attributes in the Q3-Matrix for the test item number 7, number 8 and number 9.

### The Linear Equation System (Analysis for the Q4-Matrix)

The Q4-Matrix was established by 5 test items namely the test item number 10, 11, 12, 19 and 20. The five attributes that established the Q4-Matrix were (A10) factorization, (A11) one-variable linear equation system, (A12) one-variable linear equation system narrative test item, (A19) two-variable linear equation system and (A20) two-variable linear equation system narrative test item. For a better description, the researcher would display the items and the attributes that established the Q4-Matrix in the Table 5.

Table 5. The Items and the Attributes that Established the Q4-Matrix

Item Number	Attribute				
	A10	A11	A12	A19	A20
10	1	0	0	0	0
11	0	1	0	0	0
12	0	0	1	0	0
19	0	0	0	1	0
20	0	0	0	0	1

The 5 attributes that established the Q4-Matrix, based on Table 5, generated 32 latent classes. The percentage of the latent classes that had been generated by the 5 attributes were displayed in the Figure 4.

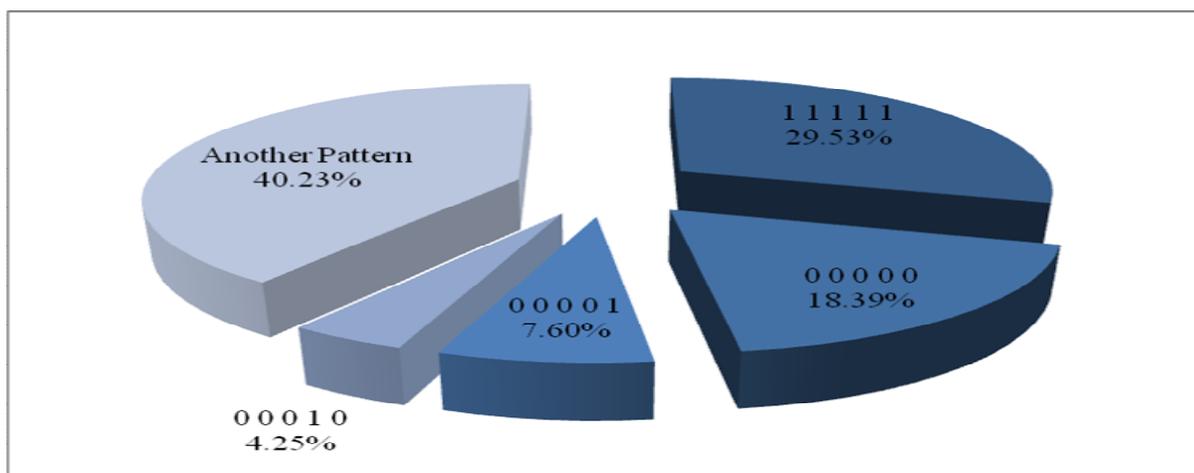


Figure 4: The Percentage of Latent Classes in Accordance with the Attributes Establishing the Q4-Matrix

Based on Figure 4, the most dominant percentage was 29.53% and was found in the latent class 1 1 1 1 1. The percentage of latent class 1 1 1 1 1 that had been equal to 29.53% showed that 29.53% of the students had mastered all of the attributes in the Q4-Matrix. Since there were only 29.53% of the students who had mastered all of the attributes in the Q4-Matrix, there were remaining 70.47% of the students who had not mastered at least one of the attributes in the Q4-Matrix. In the Figure 4, the percentage of the latent class 0 0 0 0 0, 0 0 0 0 1 and 0 0 0 1 0 was 18.39%, 7.60% and 4.25% respectively.

The percentage of the latent class 0 0 0 0 0, 0 0 0 0 1 and 0 0 0 1 0 had been the most dominant percentage after that of the latent class 1 1 1 1 1. The percentage of the latent class 0 0 0 0 0 showed that in this latent class the students had not mastered all of the attributes that established the Q4-Matrix. Then, the percentage of the latent class 0 0 0 0 1 showed that the students had only mastered one attribute namely (A20) the ability of completing the two-variable linear equation system narrative test item. Next, the percentage of the latent class 0 0 0 1 0 showed that the students had

only mastered one attribute namely (A19) the capacity of completing the one-variable linear equation system test item. From the percentage of the latent class 0 0 0 0 0, 0 0 0 0 1 and 0 0 0 1 0 showed that there were three attributes that the students from the three classes had not mastered altogether. These attributes were (A10) the factorization ability, (A11) the concept of one-variable linear equation system and (A12) the concept of one-variable linear equation system narrative text item. Therefore, the most dominant students' difficulties in accordance with the attributes that established the Q4-Matrix were the lack of mastering the factorization ability, the lack of understanding toward the concept of one-variable linear equation system and the lack of understanding toward the concept of one-variable linear equation system narrative text items.

**The Sets (Analysis for the Q5-Matrix)**

The Q5-Matrix was established by two items namely the test item number 13 and 14. The attributes that established the Q5-Matrix were (A13) partial sets and (A14) intersection of sets. Both of the attributes that established the Q5-Matrix had been the attributes of the test item number 13 and 14 and these attributes would be displayed in Table 6.

Table 6: The Items and the Attributes that Established the Q5-Matrix

Item Number	Attribute	
	A13	A14
13	0	1
14	1	0

According to Table 6, there were two attributes that established the Q5-Matrix. The two attributes would generate 4 latent classes. The percentage of each latent class would be used for identifying the difficulties in learning mathematics according to the attributes of Q5-Matrix that had not been master. For a better description, the researcher would like to display the most dominant percentage from each latent class in the Figure 5.

Based on Figure 5, the most dominant percentage was 44.74% and was found in the latent class 0 0. The percentage showed that 44.74% of the students had not mastered all of the attributes in the Q5-Matrix. The attributes that established the Q5-Matrix were the ones that had been found in the test item number 13, namely (A13), and the test item number 14, namely (A14). In order to provide the correct answer for the test item number 13, the students should be able to count the number of partial sets. Then, in order to provide the correct answer for the test item number 14 the students should be able to determine the members and the intersections of sets. According to the attributes in the Q5-Matrix, most of the students had not been able to determine the number of partial sets as well as the members and the intersections of sets.

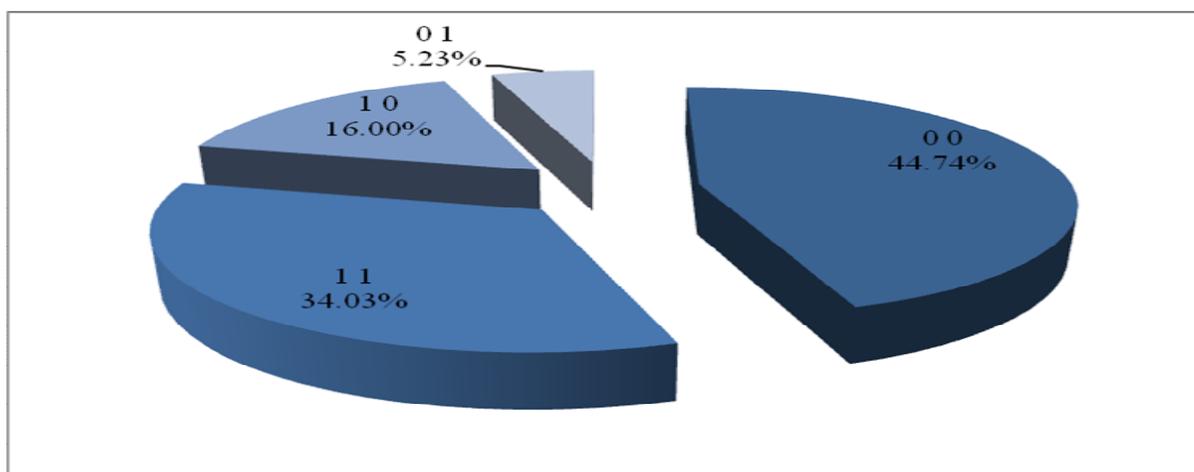


Figure 5: The Percentage of Latent Class in Accordance with the Attributes Establishing the Q5-Matrix

**The Line Function and Equation (Analysis for the Q6-Matrix)**

The Q6-Matrix was established by four items namely the test item number 15, 16, 17 and 18. Then, the attributes that established the Q6-Matrix were (A15) function, (A16) function graphic, (A17) line equation and (A18) determining the position of a point in the straight line. The items and the attributes that established the Q6-Matrix would be presented in the Table 7.

Table 7: The Items and the Attributes that Established the Q6-Matrix

Item Number	Attribute			
	A15	A16	A17	A18
15	1	0	0	0
16	0	1	0	0
17	0	0	1	0
18	0	0	0	1

The four attributes that established the Q6-Matrix, based on Table 7, generated 16 latent classes. The percentage of the latent classes that had been generated by the four attributes would be presented in the Figure 6.

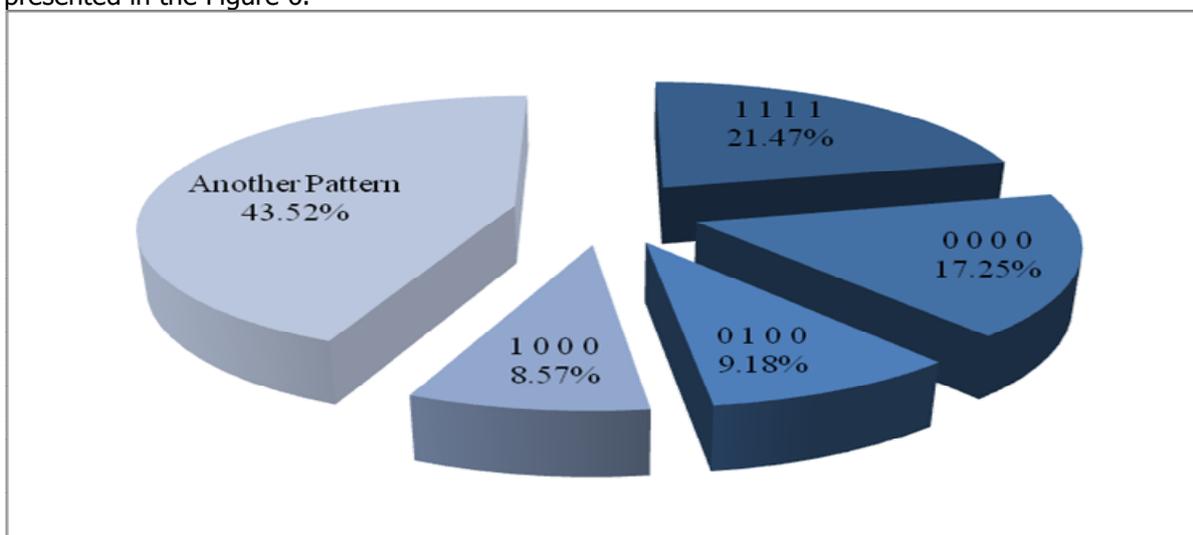


Figure 6: The Percentage of Latent Class According to the Attributes Establishing the Q6-Matrix

Based on Figure 6, the most dominant percentage was equal to 21.47% and was found in the latent class 1 1 1 1. The percentage of latent class 1 1 1 1 that had been equal to 21.47% showed that the students had mastered all of the attributes in the Q6-Matrix. Since there had been 21.47% students who had mastered the attributes in the Q6-Matrix, the remaining 78.53% students had not mastered at least one of the attributes in the matrix. In Figure 6, the percentage of the latent class 0 0 0 0, 0 1 0 0 and 1 0 0 0 was 17.25%, 9.18% and 8.57% respectively.

The percentage of the latent class 0 0 0 0, 0 1 0 0 and 1 0 0 0 was the second dominant percentage after that of the latent class 1 1 1 1. The latent class 0 0 0 0 showed that the students had not mastered all of the attributes that established the Q6-Matrix. Then, the latent class 0 1 0 0 showed that the students had only mastered one of the attributes that established the Q6-Matrix namely (A16) function graphic. Next, the latent class 1 0 0 0 showed that the students had mastered only one of the attributes that established the Q6-Matrix. From the three latent classes, it was apparent that there had been two attributes that had not been mastered by the students altogether in the same time. These attributes were (A17) bee line equation and (A18) determining the position of a point in the straight line. Therefore, the most dominant learning difficulties that the students had, according to the attributes that established the Q6-Matrix, were the capacity of determining the straight line equation and the ability of determining the position of a point in the straight line.

**The Planes (Analysis for the Q7-Matrix)**

The Q7-Matrix was established by two items namely the test item number 22 and 23. Then, the attributes that established the Q7-Matrix were (A22) planes breadth and (A23) planes periphery. Both of the attributes in the Q7-Matrix would generate four latent classes. For a better description, the researcher would like to describe the two attributes that established the Q7-Matrix in the Table 8.

Table 8: The Items and the Attributes that Established the Q7-Matrix

Item Number	Attribute	
	A22	A23
22	0	1
23	1	0

The percentage of the four latent classes, according to the number of attributes in Table 8, would be used for determining which attribute that the students had not mastered. For a better description on the percentage of each latent class, the researcher would like to display the Figure 7. According to Figure 7, it was apparent that there had been four latent classes generated from the attributes that established the Q7-Matrix. The four latent classes were 1 1, 1 0, 0 1 and 0 0. Each latent class had a percentage that had been in accordance with the results of analysis by CDM packages.

Based on Figure 6, the most dominant percentage was equal to 40.39% and was found in the latent class 1 1. The percentage of latent class 1 1 that had been equal to 40.39% showed that 40.39% students had mastered all of the attributes in the Q7-Matrix. Since there had been 40.39% students who had mastered the attributes in the Q7-Matrix, the remaining 59.61% students had not mastered at least one of the attributes in the matrix. In Figure 7, it was apparent that the latent class 0 1 had the second dominant percentage after that of the latent class 1 1 namely 39.61%. The latent class 0 1 showed that the students in the latent class 0 1 had not mastered the attributes for determining the planes (A22). Therefore, the researcher might conclude that the most dominant attribute that the students had not mastered in the Q8-Matrix had been the ability of determining the planes area.

**The Triangle Characteristics (Analysis for the Q8-Matrix)**

The Q8-Matrix was established by 5 test items namely the test item number 24, 25, 26, 27 and 28. Then, the attributes that established the Q8-Matrix were (A24) the concept of congruent triangle, (A25) the concept of triangle comparison, (A26) the concept of triangle comparison in the form of statement, (A27) the concept of angles in a triangle and (A28) drawing the dividing/height line. For a better description, the researcher would like to present the items and the attributes that established the Q8-Matrix in the Table 9.

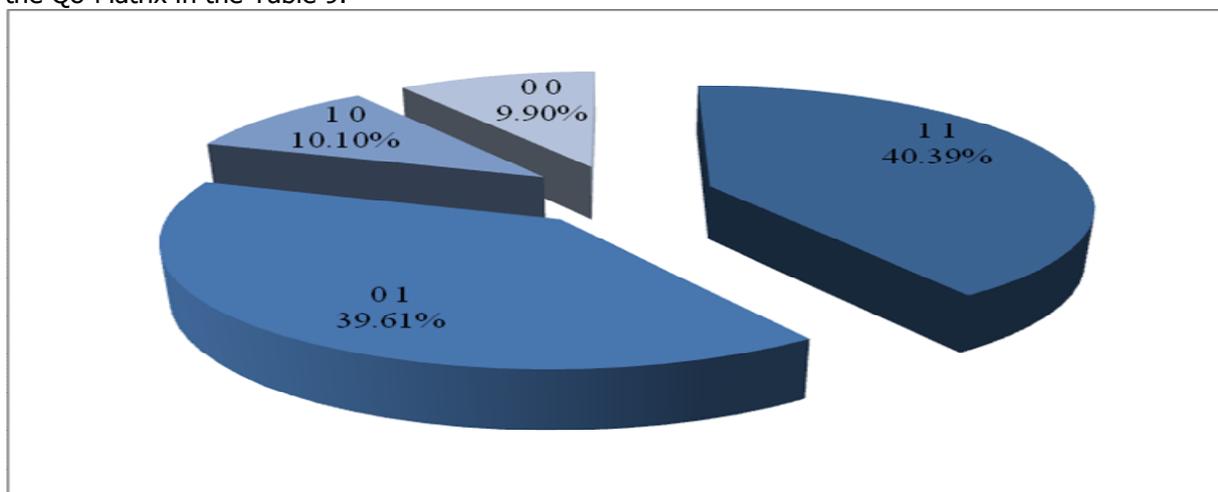


Figure 7: The Latent Class Percentage According to the Attributes Establishing the Q7-Matrix

Table 9: The Items and the Attributes that Established the Q8-Matrix

Item Number	Attribute				
	A24	A25	A26	A27	A28
24	1	0	0	0	0
25	0	1	0	0	0
26	0	0	1	0	0
27	0	0	0	1	0
28	0	0	0	0	1

The 5 attributes that established the matrix, based on Table 9, generated 32 latent classes. The percentage that had been generated by each latent class would be displayed in the Figure 8. In Figure 8, the researcher presented the most dominant percentage of the latent class that had been attained from the results of analysis.

The most dominant percentage in the above figure was 22.49% and was found in the latent class 1 1 1 1 1. The most dominant percentage found in the latent class 1 1 1 1 1 showed that 22.49% students had mastered all of the attributes that established the Q8-Matrix. Since there had been only 22.49% students who mastered all attributes in the matrix, the remaining 77.51% students had not mastered at least one of the attributes that established the Q8-Matrix. Based on the percentage in Figure 8, 13.75% students had not mastered all attributes in the Q8-Matrix namely in the latent class 0 0 0 0 0. The percentage of the latent class was 7.52% which implied that 7.52% students had mastered one of the attributes namely drawing the dividing/height line.

Still according to Figure 8, the percentage of the latent class 1 0 0 0 0 was equal to 6.63% which implied that 6.63% students had only mastered one of the attributes namely (A24) congruent triangle. The other three latent classes, namely the latent class 0 0 0 0 0, 0 0 0 0 1 and 1 0 0 0 0, had been the latent classes that had the second highest percentage after the latent class 1 1 1 1 1. From the three latent classes, the researcher found that the students in the three latent classes had not mastered the following attributes (A25) triangle comparison, (A26) the triangle comparison in the form of statement; and (A27) the angles within a triangle. Therefore, the most dominant attributes that the students had not mastered were the concept of triangle comparison, the triangle comparison in the form of statement and the angles within a triangle.

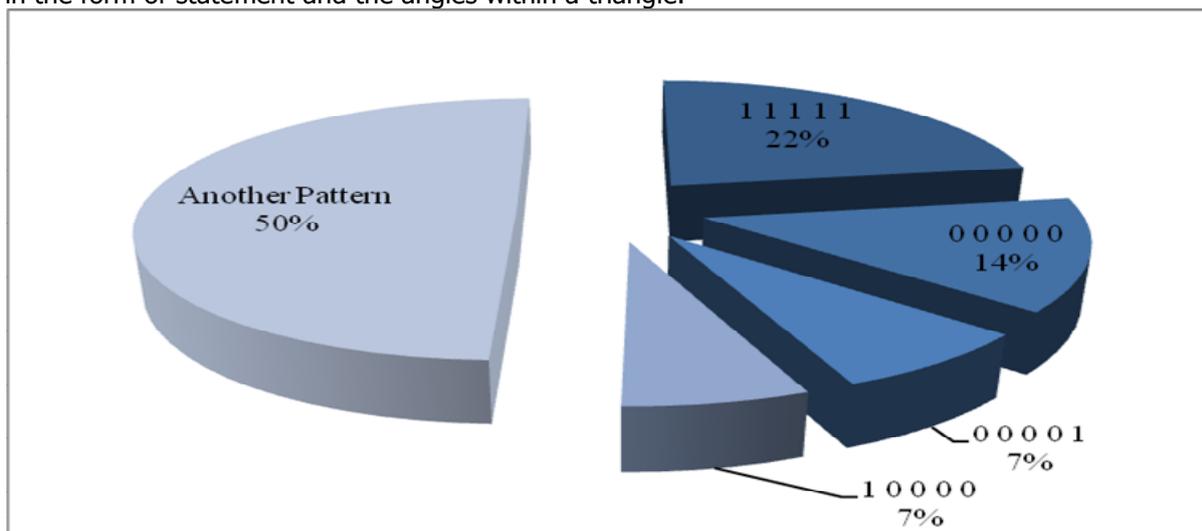


Figure 8: The Percentage of Latent Class According to the Attributes Establishing the Q8-Matrix

**The Circle and Its Tangent (Analysis for the Q9-Matrix)**

The Q9-Matrix was established by two test items namely the test item number 29 and 30. The attributes that established the Q9-Matrix were (A29) the concept of arch length and (A30) the

concept of inner allying tangent. The items and the attributes that established the Q9-Matrix would be presented in the Table 10.

Table 10: The Items and the Attributes that Established the Q9-Matrix

Item Number	Attribute	
	A29	A30
29	0	1
30	1	0

According to Table 10, there were two attributes that established the Q9-Matrix. The two attributes would generate 4 latent classes. The analysis by means of CDM packages in the R program would generate the percentage for each latent class. The percentage of each latent class would be used for identifying the difficulties in learning mathematics based on the attributes in the Q9-Matrix that the students had not mastered. For a better description, the researcher would like to display the most dominant percentage in the Figure 9.

Based on Figure 9, the most dominant percentage was found in the latent class 1 1 namely 42.35%. The most dominant percentage showed that 42.35% students had mastered all attributes in the Q9-Matrix. The second most dominant percentage was 35.58% and was found in the latent class 0 0. The second most dominant percentage showed that 35.58% students had not mastered all attributes in the Q9-Matrix. Therefore, the researcher might conclude that the most dominant attribute the students had not mastered in the Q9-Matrix had been the concept of arch length and the concept of inner tangent line.

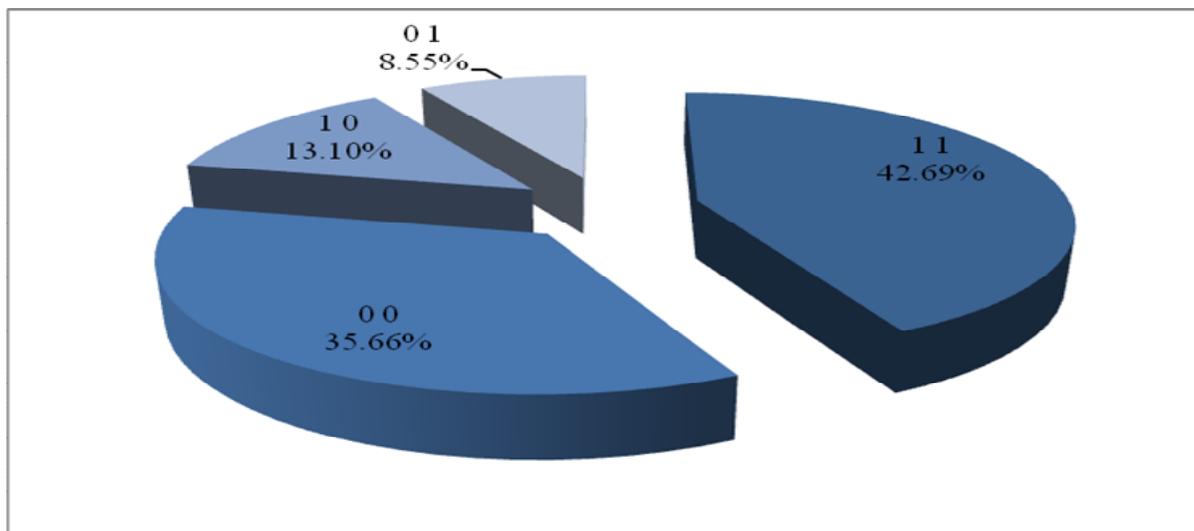


Figure 9: The Percentage of Latent Class According to the Attributes Establishing the Q9-Matrix

### The Solids (Analysis for the Q10-Matrix)

The Q10-Matrix was established by five items namely the test item number 31, 32, 33, 34 and 35. Then, the attributes that established the Q10-Matrix were (A31) the prism characteristics, (A32) the cube net, (A33) the prism volume, (A34) the breadth of solids surface (pyramid) and (A35) the breadth of solids surface (cone). The items and the attributes that established the Q10-Matrix would be presented in Table 11.

The percentage of the 32 latent classes, according to the number of the attributes in Table 11, would be used for determining which attribute that the students had not mastered. In order to define the percentage of each latent class, the researcher would like to display the results of percentage

calculation in Figure 10. The percentage of each latent class had been in accordance with the results of analysis by CDM packages.

Table 11: The Items and the Attributes that Established the Q10-Matrix

Item Number	Attribute				
	A31	A32	A33	A34	A35
31	1	0	0	0	0
32	0	1	0	0	0
33	0	0	1	0	0
34	0	0	0	1	0
35	0	0	0	0	1

Based on Figure 10, the most dominant percentage was 31.63% and was found in the latent class 1 1 1 1 1. The most dominant percentage showed that 31.63% students had mastered all attributes in the Q10-Matrix. Since there had been 31.63% students who mastered all attributes in the matrix, the remaining 68.37% students had not mastered at least one of the attributes that established the Q10-Matrix. Still based on Figure 10, 11.97% students had only mastered one of the attributes in the matrix and the percentage was found in the latent class 0 1 0 0 0. The students in the latent class 0 1 0 0 0 only mastered one attribute namely (A32) the concept of cube net. The percentage of the students who had not mastered all attributes in the Q10-Matrix was equal 10 8.95% and was found in the latent class 0 0 0 0 0. Then, the percentage of the latent class 0 1 1 0 0 was equal to 7.12% which implied that 7.12% students only mastered two attributes namely (A32) the cube net and (A33) the prism volume.

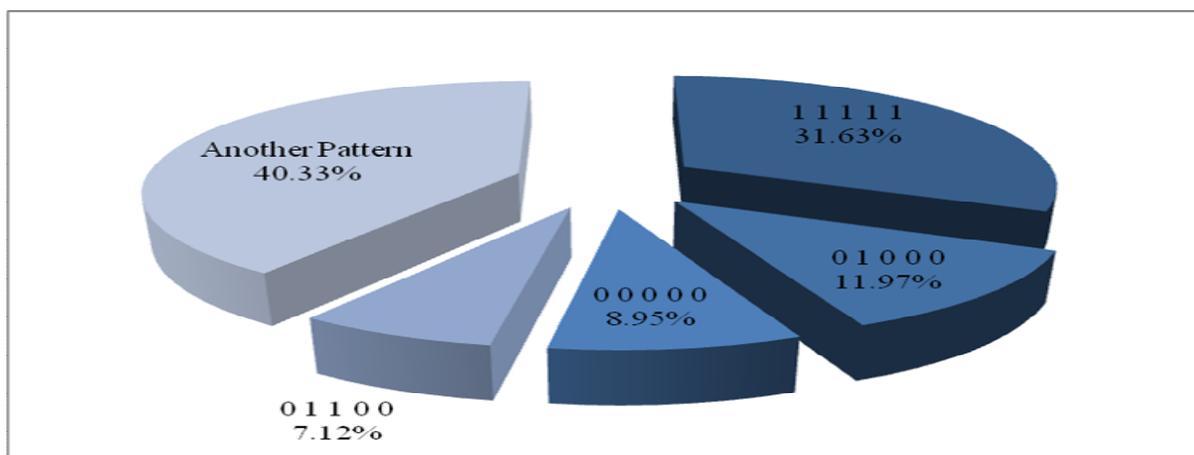


Figure 10: The Percentage of Each Latent Class According to the Attributes Establishing the Q10-Matrix

The three latent classes, namely the latent class 0 1 0 0 0, 0 0 0 0 0 and 0 1 1 0 0 had the highest percentage after that of the latent class 1 1 1 1 1. From the three latent classes, it was apparent that the students in these three classes had not mastered the following attributes: (A31) the prism characteristics, (A34) the breadth of solids surface (pyramid) and (A35) the breadth of solids surface (cone). Therefore, the attributes that the students had not mastered dominantly were the prism characteristics, the breadth of solids surface (pyramid) and the breadth of solids surface (cone).

### The Opportunity and Statistics (Analysis for the Q11-Matrix)

The Q11-Matrix was established by five items namely the test item number 36, 37, 38, 39 and 40. Then, the attributes that established the Q1-Matrix were (A36) the median, (A37) the mean, (A38) narrative text items about mean, (A39) the graphic reading and (A40) the opportunity. The test items and the five attributes that established the Q11-Matrix would be clearly presented in the Table 12.

Table 12: The Items and the Attributes that Established the Q11-Matrix

Item Number	Attribute				
	A36	A37	A38	A39	A40
36	1	0	0	0	0
37	0	1	0	0	0
38	0	0	1	0	0
39	0	0	0	1	0
40	0	0	0	0	1

According to Table 12, there were five attributes that established the Q11-Matrix. The five attributes would generate 32 latent classes. The analysis by means of CDM packages in the R program would generate the percentage of each latent class. The percentage of each latent class would be used for identifying the difficulties in learning Mathematics according to the attributes of the Q11-Matrix that the students had not mastered. For a better description, the researcher would like to display the most dominant percentage for each latent class in the Figure 11.

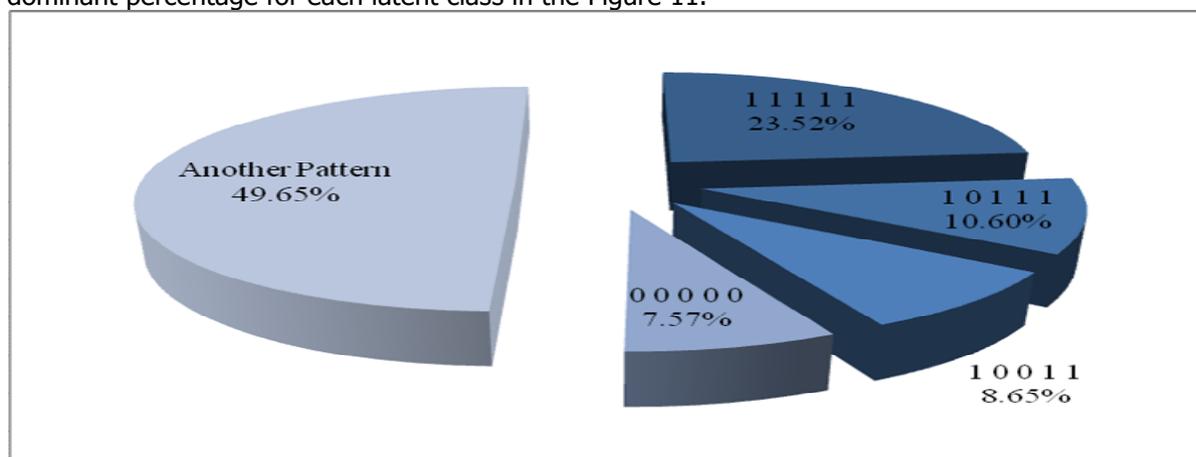


Figure 11: The Percentage of Latent Class According to the Attributes Establishing the Q11-Matrix

Based on the Figure 11, the most dominant percentage was 23.52% and was found in the latent class 1 1 1 1 1. The most dominant percentage showed that 23.52% students had mastered all attributes in the Q11-Matrix. The second most dominant percentage was 10.60% and was found in the latent class 1 0 1 1 1. The second most dominant percentage showed that 10.60% students had not mastered only one of the attributes in the Q11-Matrix namely (A37) the concept of mean. The following dominant percentage was 8.65% and was found in the latent class 1 0 0 1 1. Such percentage showed that 8.65% students had not mastered two attributes namely (A37) the mean and (A38) the mean narrative text. In addition, based on Figure 11 7.57% students had not mastered all attributes in the Q11-Matrix and the percentage was found in the latent class 0 0 0 0 0. The three latent classes that had the most dominant percentage after that of the latent class 1 1 1 1 1 were the latent class 1 0 1 1 1, 1 0 0 1 1 and 0 0 0 0 0. The students in these three latent classes altogether had not mastered the following attribute: (A37) the concept of mean. Therefore, the attribute that the students had not mastered dominantly in the Q11-Matrix was the concept of mean.

## DISCUSSIONS AND CONCLUSIONS

Paying attention to the slip and guessing parameter from the DINA model, the researcher found the test items that had big slip parameter value namely the test item number 4 and 5. Then, the test items that had the big guessing parameter value were the test item number 22 and 32. The test item number 4 was related to the attribute of square root operation, while the test item number 5 was related to the attribute of rationalizing the denominator that contained the square root form. Within the test item number 4 and 5, the slip occurred because the students had mastered all the necessary

capacities for completing both test items correctly but they slipped themselves and they provided the incorrect answer. Then, the test item number 22 was related to calculating the planes breadth in combination with the round symmetry while the test item number 32 was related to the attributes of cube net. Within the test item number 22 and 32, the students actually had not mastered all necessary attributes but they had been able to guess the right answer of both test items. The relationship between the slip and guessing parameter with regards to the item discriminative capacity had been in line with the opinion of Rupp, Templin & Henson (2010).

Based on the results of analysis, the researcher would like to conclude that the students had several difficulties in completing the national examination test items. These difficulties were caused by the fact that the students had not been able to understand the narrative text items and to determine the results of fraction operation, to determine the exponentials, to determine the results of fraction operation and of rationalizing the square root-type denominator, to perform factorization, to understand the one-variable and two-variable linear equation system narrative text item, to determine the number of partial sets, to determine the number of sets member, to determine the intersection of sets, to determine the straight line equation, to determine the position of a point in the straight line, to determine the planes area, to understand the concept of triangle comparison, to understand the concept of arch length, to understand the concept of inner allying tangent, to understand the prism characteristics, to understand the area of solid surface and to understand the concept of mean.

Regarding the difficulties in understanding the narrative text items, the results of the study were in accordance with those of Kusaeri (2014). In the recent study, the students' difficulties that the researcher might describe had been overwhelming. The results might be followed up by the Mathematics teacher or by the schools through the efforts of improving the learning process related to the learning materials. This aspect is in accordance with the function of assessment in its relationship to the instruction decision by benefitting the results of assessment for improving the learning process (Reynold, Livingstone & Wilson, 2010).

Paying attention to the results of the study, the researcher senses that the future studies are still necessary. In order to view the strengths and the weaknesses as having been displayed by the results of diagnostic test, there should be a profile of students' capacity for each of their sub-skills as having been suggested by Sun & Suzuki (2013). The results of DINA-type CDA for diagnosing the students' difficulties might take the form of web flowchart for all sub-skills so that the researcher might figure out the follow-up actions that each student should take. Detecting the misspecification of the Q-Matrix and the importance of developing the CAT cognitive diagnostic might also be pursued especially for detecting the students' difficulties during the national examination, as having been conducted by Romero, Ordonez & Ponsoda (2014) for the detection of Q-Matrix misspecification and by Huebner (2010) for the CAT cognitive diagnostic development. The CAT will ease the teachers and the students to follow up the diagnostic results in order to improve the students' capacity.

#### **BIODATA AND CONTACT ADDRESSES OF AUTHORS**



RETNAWATI is a lecturer and researcher in Mathematics Department, Mathematics and Science Faculty, Yogyakarta State University, Indonesia. Her interest of researches are teaching methods and assessment in mathematics education, measurement in education, and psychometric.

Dr. Heri RETNAWATI  
Mathematics and Science Faculty, Yogyakarta State University  
Yogyakarta- INDONESIA  
E. Mail: [heri\\_retnawati@uny.ac.id](mailto:heri_retnawati@uny.ac.id)

## REFERENCES

- Basokcu, T.O., Ogretmen, T., Kelecioğlu, H. (2013). Model data fit comparison between DINA and G-DINA in cognitive diagnostic models. *Education Journal*, 2(6): 256-262. DOI: 10.11648/j.edu.20130206.18.
- Chen, J., de la Torre, J., Zhang, Z. (2013). [Relative and Absolute Fit Evaluation in Cognitive Diagnosis Modelling.](#) *Journal of Educational Measurement*, 50(2), 123-140. DOI: 10.1111/j.1745-3984.2012.00185.
- de la Torre, J. (2008). DINA model and parameter estimation: A didactic. *Journal of Educational and Behavioral Statistics*, 39 (1), 115-130.
- de la Torre. (2015). Analysis of clinical data from cognitive diagnosis modelling framework. *Measurement and Evaluation in Counseling and Development*, 27(2), 1-16.
- Gierl, M.J., Leighton, J.P., & Hunka, S.M. (2004). The attribute hierarchy method for cognitive assessment: A variation on Tatsuoaka's rule-space approach. *Journal of Educational Measurement*, 41(3), 205-237
- Gierl, M.J., Zhou, J., & Cui, Y. (2009). Attribute reliability in cognitive diagnostic assessment. *Journal of Educational Measurement*, 46, 293-313.
- Gierl, M.J., & Wang, C. (2011). Using the attribute hierarchy method to make diagnostic inferences about examinees' cognitive skills in critical reading. *Journal of Educational Measurement*, 48(2), 165-187.
- Gronlund, Norman E. (2011). *Measurement and Assessment in Teaching* 6<sup>th</sup> Edition. Pearson: Education Inc.
- Huebner, Alan, (2010). An Overview of Recent Developments in Cognitive Diagnostic Computer Adaptive Assessments. *Practical Assessment, Research & Evaluation*, 15(3). Retrieved from <http://pareonline.net/getvn.asp?v=15&n=3>.
- Kusaeri. (2012). Menggunakan Model Dina Dalam Pengembangan Tes Diagnostik Untuk Mendeteksi Salah Konsepsi [Dina in Development Model Using Diagnostic Tests For Detecting Wrong Conception]. *Jurnal Penelitian dan Evaluasi Pendidikan*, 16(1), 281-306.
- Leighton, J.P., & Gierl, M.J. (2007). *Cognitive diagnostic assessment for education*. New York: Cambridge University Press.
- Leighton, J.P., & Gierl, M.J. (2007). Defining and evaluating models of cognition used in educational measurement to make inferences about examinees thinking processes. *Educational Measurement: Issues and Practice*, 26(5), 3-16.
- Mehrens, W.A., & Lehmann, I.J. (1984). *Measurement and evaluation in education and psychology*. New York: Holt, Rinehart and Winston. Inc
- Nitko, A.J. & Brookhart, S.M. (2007). *Educational assessment of students*. Boston: Pearson Merrill Prentice Hall.
- Peraturan Pemerintah Republik Indonesia No. 19 tahun 2005 tentang Standar Nasional Pendidikan [The Government Regulation of Republik Indonesia Number 19 Year 2005 about Education National Standard].

Ravand, Hamdollah & Robitzsch, Alexander (2015). Cognitive Diagnostic Modeling Using **R**. *Practical Assessment, Research & Evaluation*, 20(11). Available online:

<http://pareonline.net/getvn.asp?v=20&n=11>

Reynolds, C.R., Livingstone, R.B., & Wilson, V. (2010). *Measurement and assessment in education* (2<sup>nd</sup>ed.). London: Pearson.

Roberts, M.R. & Gierl, M.J. (2010). Developing score reports for cognitive diagnostic assessment. *Educational Measurement: Issues and Practice*, 29(3), 25-38.

Romero, S.J., Ordoñez, X.G., Ponsoda, V. (2014). Detection of Q-matrix misspecification using two criteria for validation of cognitive structures under the Least Squares Distance Model. *Psicológica* (2014), 35, 149-169.

Rupp, A.A., Templin, J. & Henson, R.A. (2010). *Diagnostic measurement: theory, methods and applications*. New York: The Guilford Press.

Sun, Y. & Suzuki, M. (2013). Diagnostic assessment for improving teaching practice. *International Journal of Information and Education Technology*, 3(6), 607-610. DOI: 10.7763/IJiet.2013.V3.345

Tatsuoka, K. (2009). *Cognitive assessment an introduction to the rule space method*. New York: Taylor & Francis Group.

Thorndike, R.L., & Hagen, E.P. (2005). *Measurement and evaluation in psychology and education*. New York: John Wiley & Sons, Inc.

Wang, C. (2011). Using the attribute hierarchy method to make diagnostic inferences about examinees' cognitive skills in critical reading. *Journal of Educational Measurement*, 48(2), 165-187.

Zheng, Y., Gierl, M.J., & Cui, Y. (2008). Using the attribute hierarchy method to identify and interpret cognitive skills that produce group differences. *Journal of Educational Measurement*, 45 (1), 65-89.

Zhou, J., Gierl, M.J., & Cui, Y. (2009). Attribute reliability in cognitive diagnostic assessment. *Journal of Educational Measurement*, 35(2), 23-24.